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Strain from three measured stretches

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Abstract—In order to solve for the strain on a plane, three measured stretches are required. For an algebraic solution, an equation for the value of reciprocal quadratic elongation λ' in a specified direction is needed, and it may be formed directly from the matrix representation of the finite strain tensor and the direction cosines relative to a co-ordinate system. Three of these equations, one for each measured stretch are then solved simply and directly for λ'_1 and λ'_2 using matrix methods.

INTRODUCTION

THE GRAPHICAL solution of the problem of determining the strain from three stretches in a plane is well known, but an algebraic solution would also be useful. Ramsay & Huber (1983, p. 91) present a set of equations whose unknowns are λ'_1 , λ'_2 and several orientational angles. As they observe, however, the solution of these equations is not easy. Sanderson (1977) offered one approach, but there is an even simpler formulation that provides an easy solution to this problem. This note shows how to obtain an equation for λ' (reciprocal quadratic elongation) which is linear in its unknowns, and how to determine the strain from three such equations in a simple way.

THE EQUATION

The key is an equation in terms of the components of the general finite-strain tensor rather than principal values. Following Nye (1960, p. 157) we can obtain such an expression for λ' from the matrix representation of the finite-strain tensor in two dimensions

$$\lambda' = [\cos\theta' \quad \sin\theta'] \begin{bmatrix} \lambda'_{xx} & \gamma'_{xy} \\ \gamma'_{yx} & \lambda'_{yy} \end{bmatrix} \begin{bmatrix} \cos\theta' \\ \sin\theta' \end{bmatrix}. \qquad (1)$$

After expanding, and using the equality of the off-diagonal terms, we have,

$$\lambda' = \lambda'_{xx}\cos^2\theta' + 2\gamma'_{xy}\cos\theta'\sin\theta' + \lambda'_{yy}\sin^2\theta'. \qquad (2)$$

FINDING THE STRAIN

With three unknowns, we need to measure three stretches in a plane, and we must then solve three of these equations for the unknown components of the strain tensor. With the identity $\sin 2\theta' = 2\sin\theta'\cos\theta'$, the three equations are,

$$\begin{aligned} \lambda'_a &= \lambda'_{xx}\cos^2\theta'_a + \gamma'_{xy}\sin 2\theta'_a + \lambda'_{yy}\sin^2\theta'_a \\ \lambda'_b &= \lambda'_{xx}\cos^2\theta'_b + \gamma'_{xy}\sin 2\theta'_b + \lambda'_{yy}\sin^2\theta'_b \\ \lambda'_c &= \lambda'_{xx}\cos^2\theta'_c + \gamma'_{xy}\sin 2\theta'_c + \lambda'_{yy}\sin^2\theta'_c. \end{aligned} \qquad (3)$$

These three equations could, of course, be solved for the three unknowns by ordinary algebra. Matrix algebra offers a simpler, more direct approach. We now write these equations in matrix form,

$$\begin{bmatrix} \lambda'_a \\ \lambda'_b \\ \lambda'_c \end{bmatrix} = \begin{bmatrix} \theta'_{11} & \theta'_{12} & \theta'_{13} \\ \theta'_{21} & \theta'_{22} & \theta'_{23} \\ \theta'_{31} & \theta'_{32} & \theta'_{33} \end{bmatrix} \begin{bmatrix} \lambda'_{xx} \\ \lambda'_{xy} \\ \lambda'_{yy} \end{bmatrix} \qquad (4)$$

where the θ'_{ij} are the corresponding orientation factors of the three equations.

The problem of three stretched belemnites from Ramsay & Huber (1983, p. 97) will show how a solution may be obtained. First, we need a co-ordinate system to describe the deformed state. Any arbitrary set of axes will do. We take the x' axis to be horizontal and the orientation of each of the three belemnites is then defined by the angle θ' it makes with this direction (Fig. 1). The stretch and orientation associated with each of the three lines are then,

$$\begin{aligned} S_a &= 1.56, \quad \theta'_a = +082° \\ S_b &= 1.82, \quad \theta'_b = -126° \\ S_c &= 2.38, \quad \theta'_c = +014°. \end{aligned}$$

Next, we form the matrix equation for this specific problem

$$\begin{bmatrix} 0.41091 \\ 0.30190 \\ 0.17654 \end{bmatrix} = \begin{bmatrix} 0.01937 & 0.27564 & 0.98063 \\ 0.34549 & 0.95106 & 0.65451 \\ 0.94147 & 0.46947 & 0.05853 \end{bmatrix} \begin{bmatrix} \lambda'_{xx} \\ \gamma'_{xy} \\ \lambda'_{yy} \end{bmatrix}. \quad (5)$$

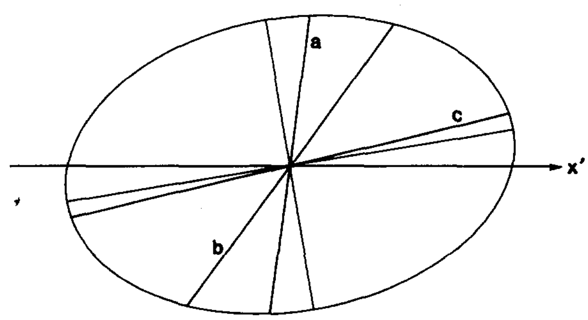

Fig. 1. Strain ellipse from three stretched lines in a plane (data from Ramsay & Huber 1983, p. 97).

In order to solve this equation, we premultiply both sides by the inverse of the 3×3 matrix (forming this inverse is a standard technique in matrix algebra; see, for example, Gere & Weaver 1965, p. 55). With this inverse we then have,

$$\begin{bmatrix} \lambda'_{xx} \\ \lambda'_{xy} \\ \lambda'_{yy} \end{bmatrix} = \begin{bmatrix} 0.44963 & -0.79387 & 1.34424 \\ -1.06503 & 1.64781 & -0.58278 \\ 1.31023 & -0.44749 & 0.13726 \end{bmatrix} \begin{bmatrix} 0.41091 \\ 0.30190 \\ 0.17654 \end{bmatrix}. \tag{6}$$

Performing this multiplication, we obtain the three independent components of the strain tensor. In matrix form, these are

$$\begin{bmatrix} \lambda'_{xx} & \gamma'_{xy} \\ \gamma'_{yx} & \lambda'_{yy} \end{bmatrix} = \begin{bmatrix} 0.18241 & -0.04305 \\ -0.04305 & 0.42753 \end{bmatrix}. \tag{7}$$

The eigenvalues of this matrix are the two principal reciprocal quadratic elongations, and they can be found from its characteristic equation; see, for example, Gere & Weaver (1965, p. 109). The results are

$$\lambda'_1 = 0.18 \quad \text{and} \quad \lambda'_2 = 0.43.$$

The eigenvectors give the orientation of these principal values; see, for example Gere & Weaver (1965, p. 111), and these establish the two orientational angles, which are

$$\theta'_1 = 9.7^\circ \quad \text{and} \quad \theta'_2 = -80.3^\circ.$$

These results are essentially identical to those obtained graphically by Ramsay & Huber (1983, p. 104).

CONCLUSIONS

An equation for λ' which is linear in the independent components of the finite-strain tensor is formed directly from its matrix representation and direction cosines. With three measured stretches, these three components are then easily found using matrix algebra. This same approach can be used in a variety of other problems, including problems of strain in three dimensions. All the required manipulations are easily programmed, even on a hand-held calculator. A FORTRAN 77 program is available on request.

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